I. Translation key:

a: Andy;  d: Dwight;  g: Angela;  j: Jim;  m: Michael;  o: the Office;  p: Pam;  t: Toby
Ax: x is an accountant;  Mx: x is a regional manager;  Rx: x is a raise;  Sx: x is a salesperson
Dxy: x despises y;  Ixy: x is in y;  Lxy: x loves y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Jim loves Pam.
   Ljp
2. Jim only loves Pam.
   Ljp • (x)(Ljx ⇒ x=p)
3. Only Andy and Dwight love Angela.
   Lag • Ldg • (x)[Lxg ⇒ (x=a ∨ x=d)]
4. There is only one accountant in the office.
   (∃x){Ax • Ixo • (y)[(Ay • Iy) ⇒ y=x]}
5. Only Michael would give Angela a raise.
   (∃x)(Rx • Gmxg) • (x)[Rx ⇒ (y)(Gyxg ⇒ y=m)]

III. Try these, using the key in I.

6. Michael is the only regional manager.

7. There is only one salesperson who despises Toby.

8. Only Dwight and Jim are salespeople in the office.
I. Translation key:

c: Creed; g: Angela; m: Michael; n: Jan; p: Pam; o: the Office; r: Scranton; s: Stanley; t: Toby
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Px: x is a person; Sx: x is a salesperson; Tx: x is a product
Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Everyone loves Pam.
   \[(x)(Px \supset Lxp)\]
2. Everyone except Angela loves Pam.
   Pa • ~Lap • (x)[(Px • x≠g) \supset Lxp]
3. Someone likes all employees except Toby.
   Et • (∃x){Px • ~Kxt • (y)[(Ey • y≠t) \supset Kxy]}
4. Everyone in the office except Pam resides in Scranton.
   Pp • Ipo • ~Vps • (x)[(Px • Ixo • x≠p) \supset Vxs]
5. Everyone but Creed passed a drug test.
   Pc • (x)(Dx \supset ~Pcx) • (x)[(Px • x≠c) \supset (∃y)(Dy • Pxy)]

III. Try these, using the key in I.

6. All employees are happy except Stanley.

7. No one except Michael tolerates Jan.

8. Some products are sold by all employees except Michael.
I. Translation key:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>Creed</td>
</tr>
<tr>
<td>d</td>
<td>Dwight</td>
</tr>
<tr>
<td>j</td>
<td>Jim</td>
</tr>
<tr>
<td>m</td>
<td>Michael</td>
</tr>
<tr>
<td>n</td>
<td>Jan</td>
</tr>
<tr>
<td>p</td>
<td>Pam</td>
</tr>
<tr>
<td>r</td>
<td>the Scranton branch</td>
</tr>
<tr>
<td>u</td>
<td>the Utica branch</td>
</tr>
<tr>
<td>Ax</td>
<td>x is an accountant</td>
</tr>
<tr>
<td>Bx</td>
<td>x is a branch</td>
</tr>
<tr>
<td>Ex</td>
<td>x is an employee</td>
</tr>
<tr>
<td>Ox</td>
<td>x is an office</td>
</tr>
<tr>
<td>Sx</td>
<td>x is a salesperson</td>
</tr>
<tr>
<td>Bxy</td>
<td>x is bigger than y</td>
</tr>
<tr>
<td>Hxy</td>
<td>x has y</td>
</tr>
<tr>
<td>Ixy</td>
<td>x is in y</td>
</tr>
<tr>
<td>Mxy</td>
<td>x is smaller than y</td>
</tr>
<tr>
<td>Nxy</td>
<td>x is nicer than y</td>
</tr>
<tr>
<td>Zxy</td>
<td>x is lazier than y</td>
</tr>
<tr>
<td>Nxyz</td>
<td>x is nearer than y to z</td>
</tr>
</tbody>
</table>

II. Examine the translations below, which use the key in I.

1. Jim is a nicer salesperson than Dwight.
   \[\text{S}j \cdot \text{S}d \cdot \text{N}jd\]

2. Jim is the nicest salesperson.
   \[\text{S}j \cdot (x)[(\text{S}x \cdot x \cdot j) \supset \text{N}jx]\]

3. Utica is the smallest branch.
   \[\text{B}u \cdot (x)[(\text{B}x \cdot x \cdot u) \supset \text{M}ux]\]

4. Creed is the laziest employee in the office.
   \[\text{E}c \cdot \text{I}co \cdot (x)[(\text{E}x \cdot \text{I}xo \cdot x \cdot c) \supset \text{Z}cx]\]

5. Michael is the employee who has the biggest office.
   \[\text{E}m \cdot (\exists x)[(\text{O}x \cdot \text{H}mx) \cdot (y)[(\text{E}y \cdot y \cdot m) \supset (z)[(\text{O}z \cdot \text{H}yz) \supset \text{B}xz])}\]

III. Try these, using the key in I.

6. Scranton is the biggest branch.
   \[\text{S}r\]

7. Utica is the nearest branch to the Scranton branch.
   \[\text{U}t \cdot (x)[(\text{U}x \cdot x \cdot t) \supset \text{N}tx]\]

8. Some employee is the biggest accountant in the office.
   \[\text{E}m \cdot (\exists x)[(\text{O}x \cdot \text{H}mx) \cdot (y)[(\text{E}y \cdot y \cdot m) \supset (z)[(\text{O}z \cdot \text{H}yz) \supset \text{B}xz)]]\]
I. Translation key:

\[ j: \text{Jim}; \quad o: \text{the Office} \]
\[ Ax: \text{x is an accountant}; \quad Dx: \text{x is a drug test}; \quad Ex: \text{x is an employee}; \quad Hx: \text{x is happy}; \quad Ix: \text{x is in the office} \]
\[ Bxy: \text{x is bigger than y}; \quad Ixy: \text{x is in y}; \quad Pxy: \text{x passed y}; \quad Txy: \text{x tolerates y} \]

II. Examine the translations below, which use the key in I.

1. There is at least one accountant in the office.
   \[ (\exists x)(Ax \land Ixo) \]

2. There are at least two accountants in the office.
   \[ (\exists x)(\exists y)(Ax \land Ixo \land Ay \land Iyo \land x \neq y) \]

3. There are at least three accountants in the office.
   \[ (\exists x)(\exists y)(\exists z)(Ax \land Ixo \land Ay \land Iyo \land Az \land Izo \land x \neq y \land x \neq z \land y \neq z) \]

4. There are at least two happy employees who tolerate each other.
   \[ (\exists x)(\exists y)(Hx \land Ex \land Hy \land Ey \land x \neq y \land Txy \land Tyx) \]

5. At least three accountants passed their drug tests.
   \[ (\exists x)(\exists y)(\exists z)(Ax \land Ay \land Az \land x \neq y \land x \neq z \land y \neq z \land (\exists w)(Dw \land Pxw) \land (\exists w)(Dw \land Pyw) \land (\exists w)(Dw \land Pzw)) \]

III. Try these, using the key in I.

6. There are at least two employees bigger than Jim.

7. There are at least three employees bigger than Jim.

8. There are at least four accountants in the office.
AAPT Workshop - A Jigsaw Lesson for First-Order Logic Translations Using Identity
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August 1, 2010

Work Group: At Most

I. Translation key:

a: Andy;  d: Dwight;  g: Angela;  m: Michael;  o: the Office
Ax: x is an accountant;  Ex: x is an employee;  Mx: x is a regional manager; Px: x is a person
Axy: x is y’s assistant;  Bxy: x is bigger than y;  Hxy: x has y;  Ixy: x is in y;  Kxy: x likes y

Note: ‘At most’ statements make no existential commitments.

II. Examine the translations below, which use the key in I.

1. At most one person is Michael’s assistant.
   \[(x)(y)[(Px \land Axm \land Py \land Aym) \rightarrow x=y]\]

2. At most two employees are accountants.
   \[(x)(y)(z)[(Ex \land Ax \land Ey \land Ay \land Ez \land Az) \rightarrow (x=y \lor x=z \lor y=z)]\]

3. At most two people are Michael’s assistants.
   \[(x)(y)(z)[(Px \land Axm \land Py \land Aym \land Pz \land Azm) \rightarrow (x=y \lor x=z \lor y=z)]\]

4. There is at most one accountant in the office bigger than Dwight.
   \[(x)(y)[(Ax \land Ixo \land Bxd \land Ay \land Iyo \land Byd) \rightarrow x=y]\]

5. At most two regional managers have employees bigger than Andy.
   \[(x)(y)(z)\{[(Mx \land \exists w)(Ew \land Hxw \land Bwa) \land My \land (\exists w)(Ew \land Hyw \land Bwa) \land Mz \land (\exists w)(Ew \land Hzw \land Bwa)] \rightarrow (x=y \lor x=z \lor y=z)\}\]

III. Try these, using the key in I.

6. There is at most one accountant in the office.

7. There are at most three accountants in the office.

8. Some people like Angela, but at most two.
Solutions to the ‘Try these’ examples

Translation key for all problems on all five worksheets:

a: Andy; c: Creed; d: Dwight; g: Angela; j: Jim; m: Michael; n: Jan; o: the Office; p: Pam; r: the Scranton branch; s: Stanley; t: Toby; u: the Utica branch

Ax: x is an accountant; Bx: x is a branch; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Mx: x is a regional manager; Ox: x is an office; Px: x is a person; Rx: x is a raise; Sx: x is a salesperson; Tx: x is a product

Axy: x is y’s assistant; Bxy: x is bigger than y; Dxy: x despises y; Fxy: x farms y; Hxy: x has y; Ixy: x is in y; Lxy: x loves y; Mxy: x is smaller than y; Nxy: x is nicer than y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y; Zxy: x is lazier than y

Gxyz: x would give y to z; Nxyz: x is nearer than y to z.

Only
6. Mm • (x)(Mx ⊃ x=m)
7. (∈x){Sx • Dxt • (y)((Sy • Dyt) ⊃ y=x]}
8. Sd • Ido • Sj • Ijo • (x)[(Sx • Ixo) ⊃ (x=d ∨ x=j)]

Except
6. Es • ¬Hs • (x)((Ex • x#s) ⊃ Hs]
7. Pm • Tmn • (x)((Px • x#m) ⊃ ¬Txn]
8. Em • (∈x){Tx • ¬Smx • (y)((Ey • y#m) ⊃ Syx]}

Superlatives
6. Br • (x)[(Bx • x#r) ⊃ Brx]
7. Br • Bu • (x)[(Bx • x#u) ⊃ Nuxs]
8. (∈x){Ex • Ixo • Ax • (y)((Ay • Iyo • y#x) ⊃ Bxy]}

At least
6. (∈x)(∈y)(Ex • Ey • x#y • Bxj • Byj)
7. (∈x)(∈y)(∈z)(Ex • Ey • Ez • Bxj • Bjy • Bzj • x#y • x#z • y#z)
8. (∈x)(∈y)(∈z)(∈w)(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo • x#y • x#z • x#w • y#z • y#w • z#w)

At most
6. (x)(y)[(Ax • Ixo • Ay • Iyo) ⊃ x=y]
7. (x)(y)(z)(w)[(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo) ⊃ (x=y ∨ x=z ∨ x=w ∨ y=z ∨ y=w ∨ z=w)]
8. (∈x)(Px • Kxa) • (x)(y)(z)[(Px • Kxa • Py • Kya • Pz • Kza) ⊃ (x=y ∨ x=z ∨ y=z)]