AAPT Workshop - A Jigsaw Lesson for First-Order Logic Translations Using Identity

Russell Marcus
Department of Philosophy, Hamilton College
rmarcus1@hamilton.edu

August 1, 2010

Work Group: Only

I. Translation key:

a: Andy; d: Dwight; g: Angela; j: Jim; m: Michael; o: the Office; p: Pam; t: Toby
Ax: x is an accountant; Mx: x is a regional manager; Rx: x is a raise; Sx: x is a salesperson
Dxy: x despises y; Ixy: x is in y; Lxy: x loves y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Jim loves Pam.

   Ljp

2. Jim only loves Pam.

   Ljp • (x)(Ljx ⊃ x=p)

3. Only Andy and Dwight love Angela.

   Lag • Ldg • (x)[Lxg ⊃ (x=a ∨ x=d)]

4. There is only one accountant in the office.

   (∃x){Ax • Ixo • (y)[(Ay • Iyo) ⊃ y=x]}

5. Only Michael would give Angela a raise.

   (∃x)(Rx • Gmxg) • (x)[Rx ⊃ (y)(Gyxg ⊃ y=m)]

III. Try these, using the key in I.

6. Michael is the only regional manager.

7. There is only one salesperson who despises Toby.

8. Only Dwight and Jim are salespeople in the office.

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Work Group: Except

I. Translation key:

c: Creed; g: Angela; m: Michael; n: Jan; p: Pam; o: the Office; r: Scranton; s: Stanley; t: Toby
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Px: x is a person; Sx: x is a salesperson; Tx: x is a product
Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Everyone loves Pam.

   (x)(Px ⊃ Lxp)

2. Everyone except Angela loves Pam.

   Pa • ~Lap • (x)[(Px • x≠a) ⊃ Lxp]

3. Someone likes all employees except Toby.

   Et • (∃x){Px • ~Kxt • (y)[(Ey • y≠t) ⊃ Kxy]}

4. Everyone in the office except Pam resides in Scranton.

   Pp • Ipo • ~Vps • (x)[(Px • Ixo • x≠p) ⊃ Vxs]

5. Everyone but Creed passed a drug test.

   Pc • (x)(Dx ⊃ ~Pcx) • (x)[(Px • x≠c) ⊃ (∃y)(Dy • Pxy)]

III. Try these, using the key in I.

6. All employees are happy except Stanley.

7. No one except Michael tolerates Jan.

8. Some products are sold by all employees except Michael.

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Work Group: Superlatives

I. Translation key:

c: Creed; d: Dwight; j: Jim; m: Michael; n: Jan; p: Pam; r: the Scranton branch; u: the Utica branch
Ax: x is an accountant; Bx: x is a branch; Ex: x is an employee; Ox: x is an office; Sx: x is a salesperson
Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Mxy: x is smaller than y; Nxy: x is nicer than y; Zxy: x is lazier than y
Nxyz: x is nearer than y to z.

II. Examine the translations below, which use the key in I.

1. Jim is a nicer salesperson than Dwight.

   $Sj \bullet Sd \bullet Njd$

2. Jim is the nicest salesperson.

   $Sj \bullet (x)[(Sx \bullet x{\neq}j) \supset Njx]$

3. Utica is the smallest branch.

   $Bu \bullet (x)[(Bx \bullet x{\neq}u) \supset Mux]$

4. Creed is the laziest employee in the office.

   $Ec \bullet Ico \bullet (x)[(Ex \bullet Ixo \bullet x{\neq}c) \supset Zcx]$

5. Michael is the employee who has the biggest office.

   $Em \bullet (\exists x)\{(Ox \bullet Hmx) \bullet (y)\{(Ey \bullet y{\neq}m) \supset (z)[(Oz \bullet Hyz) \supset Bxz]\}\}$

III. Try these, using the key in I.

6. Scranton is the biggest branch.

7. Utica is the nearest branch to the Scranton branch.

8. Some employee is the biggest accountant in the office.

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Work Group: At Least

I. Translation key:

j: Jim; o: the Office
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Ix: x is in the office
Bxy: x is bigger than y; Ixy: x is in y; Pxy: x passed y; Txy: x tolerates y

II. Examine the translations below, which use the key in I.

1. There is at least one accountant in the office.

   $(\exists x)(Ax \bullet Ixo)$

2. There are at least two accountants in the office.

   $(\exists x)(\exists y)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet x \neq y)$

3. There are at least three accountants in the office.

   $(\exists x)(\exists y)(\exists z)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet Az \bullet Izo \bullet x \neq y \bullet x \neq z \bullet y \neq z)$

4. There are at least two happy employees who tolerate each other.

   $(\exists x)(\exists y)(Hx \bullet Ex \bullet Hy \bullet Ey \bullet x \neq y \bullet Txy \bullet Tyx)$

5. At least three accountants passed their drug tests.

   $(\exists x)(\exists y)(\exists z)[Ax \bullet Ay \bullet Az \bullet x \neq y \bullet x \neq z \neq y \neq z \bullet (\exists w)(Dw \bullet Pxw) \bullet (\exists w)(Dw \bullet Pyw) \bullet (\exists w)(Dw \bullet Pzw)]$

III. Try these, using the key in I.

6. There are at least two employees bigger than Jim.

7. There are at least three employees bigger than Jim.

8. There are at least four accountants in the office.

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Work Group: At Most

I. Translation key:

a: Andy; d: Dwight; g: Angela; m: Michael; o: the Office
Ax: x is an accountant; Ex: x is an employee; Mx: x is a regional manager; Px: x is a person
Axy: x is y's assistant; Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y

Note: 'At most' statements make no existential commitments.

II. Examine the translations below, which use the key in I.

1. At most one person is Michael's assistant.

   (x)(y)[(Px • Axm • Py • Aym) ⊃ x=y]

2. At most two employees are accountants.

   (x)(y)(z)[(Ex • Ax • Ey • Ay • Ez • Az) ⊃ (x=y ∨ x=z ∨ y=z)]

3. At most two people are Michael's assistants.

   (x)(y)(z)[(Px • Axm • Py • Aym • Pz • Azm) ⊃ (x=y ∨ x=z ∨ y=z)]

4. There is at most one accountant in the office bigger than Dwight.

   (x)(y)[(Ax • Ixo • Bxd • Ay • Iyo • Byd) ⊃ x=y]

5. At most two regional managers have employees bigger than Andy.

   (x)(y)(z){[Mx • (∃w)(Ew • Hxw • Bwa) • My • (∃w)(Ew • Hyw • Bwa) • Mz • (∃w)(Ew • Hzw • Bwa)] ⊃ (x=y ∨ x=z ∨ y=z)}

III. Try these, using the key in I.

6. There is at most one accountant in the office.

7. There are at most three accountants in the office.

8. Some people like Angela, but at most two.

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Solutions to the 'Try these' examples

Translation key for all problems on all five worksheets:

a: Andy; c: Creed; d: Dwight; g: Angela; j: Jim; m: Michael; n: Jan; o: the Office; p: Pam; r: the Scranton branch; s: Stanley; t: Toby; u: the Utica branch

Ax: x is an accountant; Bx: x is a branch; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Mx: x is a regional manager; Ox: x is an office; Px: x is a person; Rx: x is a raise; Sx: x is a salesperson; Tx: x is a product

Axy: x is y's assistant; Bxy: x is bigger than y; Dxy: x despises y; Fxy: x farms y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Mxy: x is smaller than y; Nxy: x is nicer than y; Pxy: x passed y; Rxy: x resides in y; Sxy: x sells y; Txy: x tolerates y; Zxy: x is lazier than y

Gxyz: x would give y to z; Nxyz: x is nearer than y to z.

Only

6. Mm • (x)(Mx ⊃ x=m)
7. (∃x){Sx • Dxt • (y)[(Sy • Dyt) ⊃ y=x]}
8. Sd • Ido • Sj • Ijo • (x)[(Sx • Ixo) ⊃ (x=d ∨ x=j)]

Except

6. Es • ~Hs • (x)[(Ex • x≠s) ⊃ Hs]
7. Pm • Tmn • (x)[(Px • x≠m) ⊃ ~Txn]
8. Em • (∃x){Tx • ~Smx • (y)[(Ey • y≠m) ⊃ Syx]}

Superlatives

6. Br • (x)[(Bx • x≠r) ⊃ Brx]
7. Br • Bu • (x)[(Bx • x≠u) ⊃ Nuxs]
8. (∃x){Ex • Ixo • Ax • (y)[(Ay • Iyo • y≠x) ⊃ Bxy]}

At least

6. (∃x)(∃y)(Ex • Ey • x≠y • Bxj • Byj)
7. (∃x)(∃y)(∃z)(Ex • Ey • Ez • Bxj • Byj • Bzj • x≠y • x≠z • y≠z)
8. (∃x)(∃y)(∃z)(∃w)(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo • x≠y • x≠z • x≠w • y≠z • y≠w • z≠w)

At most

6. (x)(y)[(Ax • Ixo • Ay • Iyo) ⊃ x=y]
7. (x)(y)(z)(w)[(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo) ⊃ (x=y ∨ x=z ∨ x=w ∨ y=z ∨ y=w ∨ z=w)]
8. (∃x)(Px • Kxa) • (x)(y)(z)[(Px • Kxa • Py • Kya • Pz • Kza) ⊃ (x=y ∨ x=z ∨ y=z)]